# P147 ON 3D ELECTROMAGNETIC INVERSE PROBLEMS IN THE CASE OF ARBITRARY RELIEF

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## Introduction

Inversion of electromagnetic (EM) data in geophysical prospecting involves solution of a nonlinear-operator equation of the first kind (with an implicit ill-conditioned operator). The numerical solution of such equations requires considerable expenditures of computer time. For the theoretical inverse problem (TIP) the author was able to obtain explicit equations for the electrical and magnetic fields, to develop effective algorithms for solving these equations, and construct examples of equivalent regions. A TIP is one in which the governing fields are specified explicitly, usually as the field of singular sources lying in a half-space. Solution of a TIP can be the last step of interpretation methods that first approximate observed data with the fields of singular sources. It also makes possible the construction of geologically meaningful equivalents for different classes of singular sources. The TIP equations are derived for EM fields satisfying the Helmholtz and telegrapher's equations. We constructed some numerical examples solving EM Inverse Problem (for different earth-air boundaries).

## Explicit equations for inverse problem

Assume that in a linear isotropic lower halfspace with conductivity $\sigma_1$ and permeability $\mu_1$, there is an inclusion, a body $T$ with parameters $\sigma_2$, $\mu_2$. Also assume that in the medium there are sources of generating electromagnetic fields, $H_1, E_1$ and $H_2, E_2$, outside and inside the conducting inclusion, respectively. We assume that $T$ is a 3-D region, $S$ is its boundary, $L$ is ground-air boundary, $r = \{x, y, z\}$ is the radius-vector of a point in $\mathrm{R}^3$. We have obtained the new inverse problem equations of electromagnetic fields. There are the first generation equations with explicit operators.

### ***Common case***

In this case equation have the form

$$E_1^\alpha(r',t') = \int_S \int_{-\infty}^{t'-\sqrt{\mu\varepsilon}|r-r'|} \Big\{ (n, E_1^\alpha) \nabla\Big(\tfrac{\varepsilon_1}{\varepsilon_2} G_2^t - G_1^t\Big) + \tfrac{\nabla G_2^t}{\varepsilon_2}\big[\eta + \varepsilon_1 (E_1^S, n)\big] + [n, E_1^\alpha] \times \nabla\big(G_2^t - G_1^t\big) + \tag{1}$$

$$+ [n, H^S]\mu_2 \tfrac{\partial G_2^t}{\partial t} + [n, E^s] \times \nabla G_2^t + [n, H_1^\alpha]\Big(\mu_2 \tfrac{\partial G_2^t}{\partial t} - \mu_1 \tfrac{\partial G_1^t}{\partial t}\Big)\Big\} dtds,$$

$$H_1^\alpha(r',t') = \int_S \int_{-\infty}^{t'-\sqrt{\mu\varepsilon}|r-r'|} \Big\{ (n, H_1^\alpha) \nabla\Big(\tfrac{\mu_1}{\mu_2} G_2^t - G_1^t\Big) + [n, H_1^\alpha] \times \nabla\big(G_2^t - G_1^t\big) + [n, H^s] \times \nabla G_2^t + \tag{2}$$

$$+ [n, E^S]\Big(\sigma_2^* G_2^t - \varepsilon_2 \tfrac{\partial G_2^t}{\partial t}\Big) + \tfrac{\mu_1}{\mu_2}\big(H_1^S, n\big)\nabla G_2^t + [n, E_1^\alpha]\big(\sigma_2^* G_2^t - \sigma_1^* G_1^t\big) - [n, E_1^\alpha]\Big(\varepsilon_2 \tfrac{\partial G_2^t}{\partial t} - \varepsilon_1 \tfrac{\partial G_1^t}{\partial t}\Big)\Big\} dtds$$

where $G^t_{1,2}$ is the fundamental solution for the telegraphic equation.

***Monochromatic field***

$$E_1^a(r')=\int_S\Big\{(n,E_1^a)\nabla(\tfrac{\varepsilon_1}{\varepsilon_2}G_2-G_1)+\nabla G_2[\eta+\varepsilon_1(E^S,n)]/\varepsilon_2+[n,E_1^a]\times\nabla(G_2-G_1)+$$
$$+[n,E^S]\times\nabla G_2+i\omega[n,H_1^a](\mu_2G_2-\mu_1G_1)+i\omega\mu_2[n,H^S]G_2\Big\}dS+ \tag{3}$$
$$+\int_L\Big\{[n,E_1^a]\times\nabla G_1+i\omega\mu_1[n,H_1^a]G_1\Big\}dL,$$

where $\eta=\left(\frac{\varepsilon_1}{\sigma_1^*}-\frac{\varepsilon_2}{\sigma_2^*}\right)\nabla|_S\cdot[n,H_1]$, since $(E,n)|_S=-\frac{1}{\sigma}\nabla|_S\cdot[n,H]$.

$$H_1^a(r')=\int_S\Big\{(n,H_1^a)\nabla(\tfrac{\mu_1}{\mu_2}G_2-G_1)+\tfrac{\mu_1}{\mu_2}(H^S,n)\nabla G_2+[n,H_1^a]\times\nabla(G_2-G_1)+$$
$$+[n,H^S]\times\nabla G_2+[n,E_1^a](\sigma_2^*G_2-\sigma_1^*G_1)+\sigma_2^*[n,E^S]G_2\Big\}dS+ \tag{4}$$
$$+\int_L\Big\{[n,H_1^a]\times\nabla G_1+\sigma_1^*[n,E_1^a]G_1\Big\}dL,$$

$$G_{1,2}(r'|r)=-\frac{\exp\left(ik_{1,2}^*|r-r'|\right)}{4\pi|r-r'|} \tag{5}$$

Relations (3) and (4) are the equations of the TIP for a monochromatic field (relative to the boundary $S$). The material properties of the anomalous region are assumed to be parameters; i.e. the solution of the TIP holds for various values $\sigma_2,\varepsilon_2,\mu_2$. The result is an equivalent family of bodies that generate the same electrical or magnetic field. In numerical solution of equations (3) and (4), it is possible to use the algorithm formulated in (Martyshko, 1999).

**Algorithm for solving the TIP equations in the class of stellate bodies**

Bodies stellar in relation to a certain internal point constitute a broad class important in practical terms. We shall construct a spherical system of coordinates with the center at that point.

Making in (3) a spherical substitution of the variables, we obtain

$$E_1^a(r')=\int_0^{2\pi}\int_0^{\pi}U_{\sigma_2,\varepsilon_2}(\rho(\theta,\varphi),\theta,\varphi,r')d\theta d\varphi, \tag{6}$$

where $U$ is function under the integral in (3), $\rho(\theta,\phi)$ is the right-hand side of the equation of the body surface $S$ — $\rho=\rho(\theta,\phi)$ in spherical coordinates. Representation (6) may be considered as the explicit equation relative to the function $\rho(\theta,\phi)$.

The right-hand side of the equation of the surface of the body $T$ may be represented by a double Fourier series:

$$\rho(\theta,\phi)=\sum_{k=-\infty}^{\infty}\sum_{j=-\infty}^{\infty}a_{kj}e^{i(k\theta+j\phi)}. \tag{7}$$

We shall therefore seek the solution of (6) in the form of a segment of the double Fourier series

$$\rho_{n,m}(\theta,\phi)=\sum_{k=-n}^{n}\sum_{j=-m}^{m}a_{kj}e^{i(k\theta+j\phi)}, \tag{8}$$

minimizing the functional

$$f(\gamma a)=\sum_{i\in M}\left[E_1^a(r_i')-E_1^{a,nm}(r_i';\gamma)\right]^2, \tag{9}$$

where the points $r_i'$ belong to the information carrier which in the case of TIP can be chosen; $\gamma$ is the vector of the coefficients of the function $\rho(\theta,\phi)$, for which minimization is performed; and $E_1^{a,nm}$ is the right-hand side of (6) substituting $\rho$ by $\rho_{n,m}$.

It is known that the function $\rho(\theta,\phi)$, which defines the boundary of the one-connected three-dimensional domain, must have the following property:

$$\rho(0,\phi)=const,\ \ \rho(\pi,\phi)=const. \tag{10}$$

From the condition above, the relation for the coefficients $\rho_{n,m}$ it follows that

$$\sum_k a_{kj}=0,\ \sum_{k_1} a_{k_1 j}=0, \tag{11}$$

$$j\in[-m,m],\ \ k,k_1\in[-n,n],\ \ k=\pm1,\pm3,\ldots;\ \ k_1=\pm2,\pm4,\ldots$$

We must find thus the minimum of the functional $f$ and subject to equality-type constraints. Practically, we add to the functional $f$ the functional $f_1$

$$f_1=\sum_j P_j\left(\left|\sum_k a_{kj}\right|^2+\left|\sum_{k_1} a_{k_1 j}\right|^2\right), \tag{12}$$

where $P_j$ are the penalty coefficients. Regularization of (6) is done by using an analog of smoothing first-order Tikhonov normalizer

$$\Theta(\rho)=\int_0^{2\pi}\int_0^{\pi}\left(\left|r_\theta'\right|^2+\left|r_\phi'\right|^2\right)d\theta d\phi=2\pi^2\sum_{k=-n}^{n}\sum_{j=-m}^{m}\left(k^2+j^2\right)\left|a_{kj}\right|^2. \tag{13}$$

In solving (6), the functional $f+f_1+\alpha\Theta$ was minimized, where the regularization parameter $\alpha$ was determined from the discrepancy. The Powell method was used for minimization; the integral in (6) was computed by using the formulas of a high trigonometric accuracy, which in this case are the formulas with equal weights and equidistant nodes. A sphere of a minimum radius including all the singularities $E_1^a$ was taken as an initial approximation. The solution was then defined for (6) (which includes the conductivity $\sigma_2$ as a parameter, making it possible to construct an whole family of equivalent bodies generating with different conductivities the similar field) for a certain value of $\sigma_2=\sigma_0$. At each subsequent step the body $T_{n-1}$ was taken as the initial approximation for the conductivity $\sigma^n<\sigma^{n-1}$. A sphere known a priori to contain the solution was chosen for the information carrier.

**Numerical examples**

As a result of inverse problem solving we obtain the bodies stellate relative to some point with different values of conductivity (permeability), which generated the same (electrical or magnetic) field. We have obtained some numerical examples ($\mu_1=\mu_2=\mu_0$, $\mu_0=4\pi\cdot10^{-7}\ H/m$). Figure 1 show the cross sections cut off by the coordinate plane on numerical results solving TIP for various boundaries between air and earth for function

$$E_1^a=\left\{Q_{1x}\frac{e^{ik_2r_1}}{r_1}+Q_{2x}\frac{e^{ik_2r_2}}{r_2}+Q_{3x}\frac{e^{ik_2r_3}}{r_3},\quad Q_{1y}\frac{e^{ik_2r_1}}{r_1}+Q_{2y}\frac{e^{ik_2r_2}}{r_2}+Q_{3y}\frac{e^{ik_2r_3}}{r_3},\quad Q_{1z}\frac{e^{ik_2r_1}}{r_1}+Q_{2z}\frac{e^{ik_2r_2}}{r_2}+Q_{3z}\frac{e^{ik_2r_3}}{r_3}\right\},\ E^S(P)=\left\{Q_{4x}\frac{e^{ik_2r_4}}{r_4},\ Q_{4x}\frac{e^{ik_2r_4}}{r_4},\ Q_{4x}\frac{e^{ik_2r_4}}{r_4}\right\},\ P_1,P_2,P_3\in T^+,\ P_4,P\in T^-,\ r_i=\left|PP_i\right|,$$

$i=1,2,3,4$, $P_4(0,0,10)$, $Q_{1x}=Q_{1y}=Q_{1z}=3$, $Q_{2x}=Q_{2y}=Q_{2z}=6$, $Q_{3x}=Q_{3y}=Q_{3z}=-9$, $Q_{4x}=Q_{4y}=Q_{4z}=10$, $\sigma_1/\sigma_2=1/6$, $\omega=2\pi\cdot10000$.

## References

Martyshko, P.S., 1999, Inverse Problems of Electromagnetic Geophysical Fields. “VSP”, Utrecht, The Netherlands, 123 p.

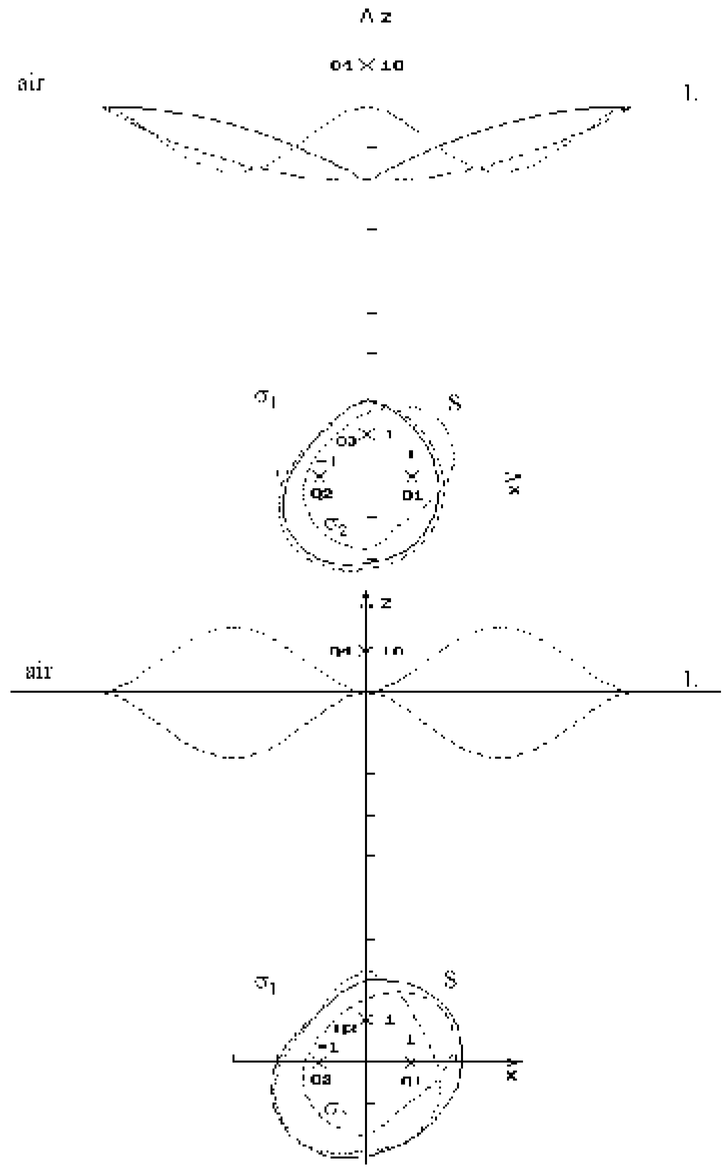


Fig. 1. S is the contour of the bodies for solutions for different boundaries between air and earth with $\sigma_1/\sigma_2 = 1/6$