P108 ON THE CONSTRUCTION OF DENSITY SECTIONS USING GRAVITY DATA

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SUMMARY
The construction process of density sections using gravity data leads to solution of linear inverse problem, which is the classical example of ill-posed problem as its solution is not unique and unstable. It is possible to choose the specific variant of density distribution if additional information is presented.
From a practical point of view a problem of allocation of gravity field sources (i.e. areas with anomalous density) located in a horizontal layer between two given depths and definition of variable density in this layer is rather interesting.
The purpose of this paper is to present new computer-based technique of gravity field sources separation using ideas from (Martyshko, 2003).

INTRODUCTION

A problem of finding "low-density" zones in horizontal layers appears during oil and gas exploration. This problem can be formulated as follows. Calculate density distribution within horizontal layer lying between two given depths. The computer program was developed for realization of gravity data interpretation method suggested in (Martyshko, 2003). We have done practical data interpretation using this program.

COMPUTER-BASED TECHNOLOGY OF GRAVITY FIELD SOURCES SEPARATION AND DENSITY CALCULATION IN HORIZONTAL LAYERS

In this part of abstract we describe a new technique for the gravity field sources separation and calculation of density distribution in a horizontal layer.
We deal with data located inside a rectangular area D (Fig. 1). Data for investigations are located usually on a non-regular grid. As far as all following procedures are developed for regular grids we have to calculate data onto a regular grid (with a uniform nodes spacing). In the procedures of calculation a problem of absence of data in some areas of the investigated region has arisen. Significant errors shouldn't arise during further processing if smooth data extrapolation onto these areas is applied.
The observed data due to sources located outside the area D too. For the purpose of reduction of this influence a special function is subtracted from measurements. This function is the Dirichlet problem solution, i.e. satisfies Laplace equation and is equal to experimental data values on the edge of the area:

$$\Delta U(x, y, z) = 0$$

$$U\Big|_{\partial D} = \Delta g_{mes}$$

where function Δg_{mes} – measured field on the edge ∂D of the area D. As far as U may have extrema only on the border ∂D we can consider U(x,y) as the values of field of sources lying outside the area (Fig. 2).
Difference $U_0=\Delta g_{mes} - U$ is equal to the field of sources lying under the area D. Values of the U_0 function are zero on the edge ∂D. Hence we have errors decreased during integration used in further procedures.
To approximate Laplace operator Δ we use standard formula

$$\Delta U=u_{m+1,n}+u_{m-1,n}+u_{m,n-1}+u_{m,n+1}-4u_{m,n},$$

where u_{ij} – is value U in the (i,j) node of the grid. In this case we get a linear system with sparse matrix with diagonal domination. Experiments show that it takes not too mush time to get solution for small (50x50) systems. Authors believe that it is possible to achieve higher speed of calculation using methods optimized for matrixes with such structure. Iteration methods also can be used for these systems.
Then we define a field from sources lying in the layer with $z\in[-H_1,-H_2]$. (Positive value of z means altitude, negative means depth). For this purpose we have to calculate fields of sources lying under $-H_1$ and $-H_2$ respectively. Then the field from sourced located between depths $-H_1$ and $-H_2$, can be calculated as difference between these fields. Following procedure was used to calculate field from sources lying under depth H. By formula

$$U(x',y',H)=\frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\frac{H}{((x-x')^2+(y-y')^2+H^2)^{3/2}}U(x,y,0)dxdy \qquad (1)$$

field is calculated to the altitude of H.
Most simple methods of calculations were used to decrease calculation time (trapeziums method takes about 0.5 sec). Then data were calculated to the depth of H using Lavrentiev's method of regularization

$$U(x',y',H)=\frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\frac{2H}{((x-x')^2+(y-y')^2+4H^2)^{3/2}}U(x,y,-H)dxdy$$

i.e. in the kernel H is changed to 2H; matrix A (coefficients of the integration) is calculated and linear system is solved with regularization

$$(A+\alpha E)U(x',y',-H)=U(x',y',H)$$

where U(x',y',-H) - unknown field. We solve this system both with iteration method and Gauss method (with leading element). Iteration method is the quickest descend method, which is given with formulae

$$x^{n+1}=x^n-\Delta^n((A+\alpha E)x^n-b)$$
$$\Delta^n=\frac{(Bx^n-b,Bx^n-b)}{(B(Bx^n-b),Bx^n-b)},$$

where $B=(A+\alpha E)$, $b=u(x',y',H)$. Precision of the solution could be estimated as residual function $(A+\alpha E)x^n - u(x',y',H)$. Results of experiments show that this method converges fast as residual function decreases 10 times during about 10 iterations.

Increase of the regularization parameter α leads to the smoothing of the solution, but larger values may cause loss of the "small details" of the field, which leads to errors in the density distribution.

At least the field $U(x', y', -H)$ is calculated onto the Earth surface using (1). To check the results obtained with iteration method we solved system with regularization applying converse matrix method. Both results were very similar, density distributions almost coincided, despite calculation of the converse matrix took 2 minutes and 30 iterations (caused decrease of the difference 100 times) required less then 10 seconds. (Fig. 4, 5)

Now we can start solving the problem of density distribution in the layer. In (Martyshko, 2003) expression of gravitational effect from horizontal layer with varying density is presented:

$$\Delta g(x', y', 0) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left(\frac{1}{\sqrt{(x-x')^2+(y-y')^2+H_1^2}} - \frac{1}{\sqrt{(x-x')^2+(y-y')^2+H_2^2}}\right)\sigma(x, y)dxdy \quad (2)$$

For inverse problem solution we must solve direct problem many times, therefore effective methods for calculation integral on the right hand of (2) are needed. Consider double integral in (2) as a sum of integrals over elementary rectangles Δx by Δy, i.e. we imagine layer separated into bars $\Delta x \cdot \Delta y \cdot (H_2 - H_1)$, each having constant density, and use 2D Gaussian integral formula with 4 nodes to achieve the highest algebraic precision degree. If $K(x, y, x', y')$ denotes kernel in (2) then double integral over elementary rectangle S can be replaced with the sum

$$\iint K(x, y, x', y')dxdy \approx \sum\sum K(x_i \pm \frac{\Delta x}{2\sqrt{3}}, y_j \pm \frac{\Delta y}{2\sqrt{3}}),$$

where (x_i, y_j) is the center of S. Then calculation of the integral in (2) comes to the multiplication of vector of density vector $\sigma(x,y)$ and matrix of integration. We calculate values of the density by solving a system with regularization $(A+\alpha E)x=u$ (Fig.6).

The advantage of this approach is the following. Integration matrix can be calculated only once and therefore every iteration of the field evaluation requires only multiplication of the vector and the matrix. The system is solved using the quickest descend method, but speed of convergence of density evaluation method is significantly lower than of calculating field down. Matrix conversion gave results very similar to ones acquired with iteration method. The influence of the regularization at the stage of density evaluation is much less than during field calculation. Increase of the parameter also leads to "smoothing" of the density picture.

Described computer technology was applied to the observed data Δg (Fig. 1) Result of the calculations (levels of density) is shown in fig.6. (All distances in km, Δg - mgl).

CONCLUSION

An original algorithm of depth gravity field sources separation, which was suggested in (Martyshko, 2003) is realized in a new computer technology. Its features are easy-to-use, openness and easy data representation. Data processing from reading of information about observed data to output of calculated values of the density is done within a single computer program. Data interpretation using this program was done with real gravity data.

Sources of gravity field lying in horizontal layer with height of 1 km (upper edge is the Earth surface) were localized and density distribution was constructed. So we have found "low density" zones. This technique can be used for purposes of oil and gas explorations.

REFERENCES

P.S.Martyshko and I.L.Prutkin. 2003. Technique for separating gravity field sources in depth. Geophysical Journal, vol. 25, N 3, p. 159-169.

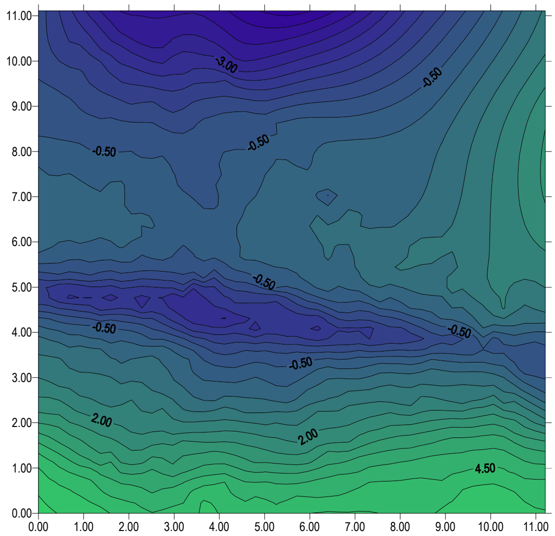

Fig. 1. Observed field

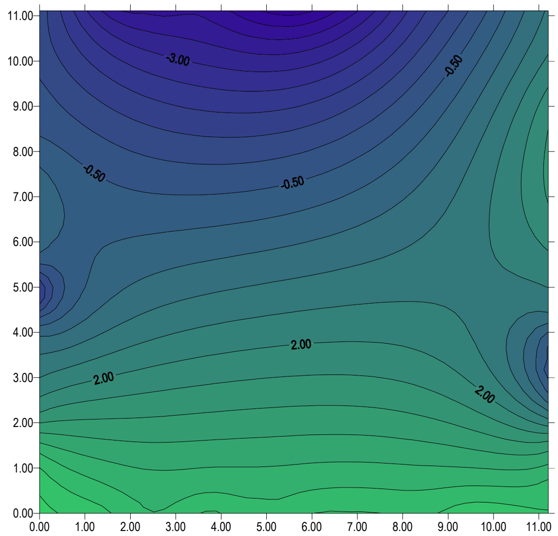

Fig. 2. Dirichlet problem solution

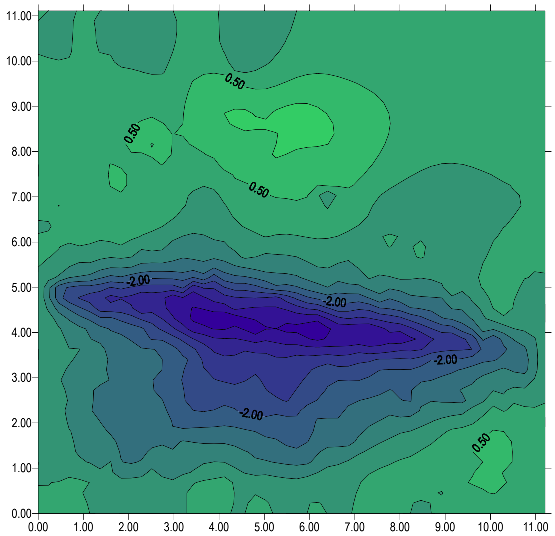

Fig. 3. Field without out-of-area sources

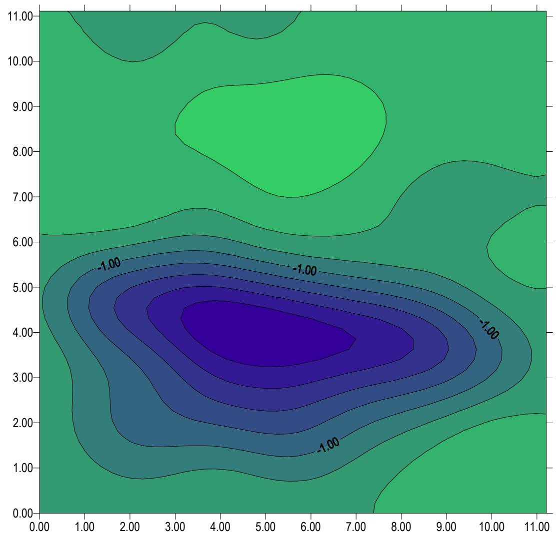

Fig. 4. Field of sources deeper than 1 km

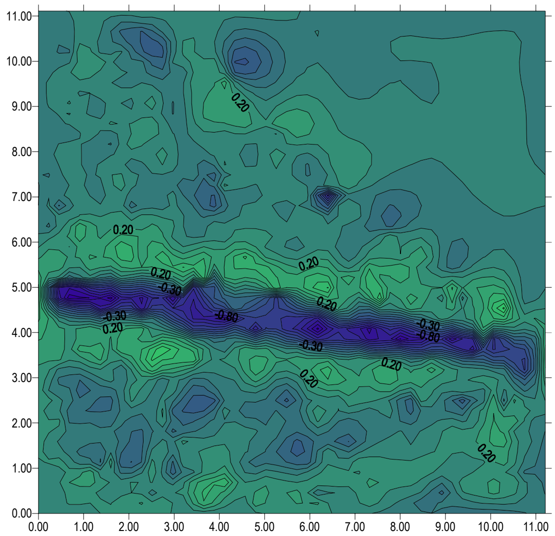
Fig. 5. Field of layer 0…1 km

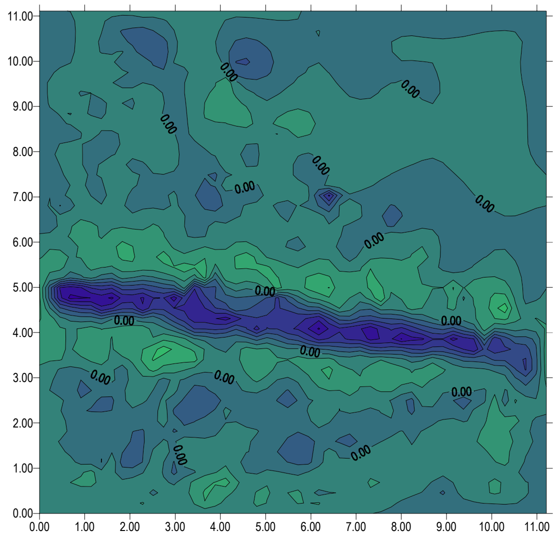
Fig. 6. Resulting density distribution

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