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ALGORITHM AND NUMERICAL EXAMPLES 3D ELECTROMAGNETIC INVERSE PROBLEM

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1 Introduction

Inversion of electromagnetic (EM) data in geophysical prospecting involves solution of a nonlinear-operator equation of the first kind (with an implicit, often ill-conditioned, operator). Numerical solution of such equations can require considerable computer time. For the theoretical inverse problem (TIP) in electrical prospecting with a dc current, however, the author was able to obtain explicit integrodifferential equations for the electrical and magnetic fields (Martyshko, 1986a,b), develop effective algorithms for solving these equations, and construct examples of equivalent regions. A TIP is one in which the governing fields are specified explicitly, usually as the field of singular sources lying in a half-space. Solution of a TIP can be the last step of interpretation methods that first approximate observed data with the fields of singular sources. It also makes possible the construction of geologically meaningful equivalents for different classes of singular sources. The TIP equations and their derivation both merit attention. TIP equations are derived for EM fields satisfying the Helmholtz and telegrapher's equations. The derivation uses representations of fields through their values and the derivatives at the boundary of the anomalous object and the Stratton-Chu formulas.

2 Explicit equations for inverse problem

Assume that in a linear isotropic lower halfspace with conductivity σ_1 and permeability μ_1, there is an inclusion, a body T with parameters σ_2, μ_2. Also assume that in the medium there are sources of generating electromagnetic fields, $(\mathbf{H}_1, \mathbf{E}_1)$ and $(\mathbf{H}_2, \mathbf{E}_2)$, outside and inside the conducting inclusion, respectively. We assume that T is a 3-D region, S is its boundary, L is ground-air boundary, $\mathbf{r}$ is the radius-vector of a point in R^3. We have obtained the new inverse problem equations of electromagnetic fields. There are the first generation equations with explisit operators.

$$\mathbf{E}_1^\alpha(\mathbf{r}') = \int_S \Big\{(\mathbf{n}, \mathbf{E}_1^\alpha)\left[\nabla\left(\frac{\varepsilon_1}{\varepsilon_2}G_2 - G_1\right)\right] + \frac{\nabla G_2}{\varepsilon_2}[\eta + (\mathbf{n}, \mathbf{E}^S)\varepsilon_1] + [\mathbf{n}, \mathbf{E}^S] \times \nabla G_2 +$$
$$+ [\mathbf{n}, \mathbf{E}_1^\alpha] \times \nabla(G_2 - G_1) + i\omega[\mathbf{n}, \mathbf{H}_1^\alpha](\mu_2 G_2 - \mu_1 G_1) + i\omega\mu_2[\mathbf{n}, \mathbf{H}^S]G_2\Big\}\, dS + \qquad (1)$$
$$+ \int_L \{[\mathbf{n}, \mathbf{E}_1^\alpha] \times \nabla G_1 + i\omega\mu_1[\mathbf{n}, \mathbf{H}_1^\alpha]G_1\}\, dL,$$

where $\eta = (\varepsilon_1/\sigma_1^* - \varepsilon_2/\sigma_2^*)\nabla_S[\mathbf{n}, \mathbf{H}_1]$, since $(\mathbf{n}, \mathbf{E}|_S) = -(1/\sigma^*)\nabla_S[\mathbf{n}, \mathbf{H}]$.

$$\begin{aligned}\mathbf{H}_1^\alpha(\mathbf{r}') = \int_S \Big\{&(\mathbf{n}, \mathbf{H}_1^\alpha)\nabla\left(\frac{\mu_1}{\mu_2}G_2 - G_1\right) + \frac{\mu_1}{\mu_2}(\mathbf{n}, \mathbf{H}^S)\nabla G_2 + [\mathbf{n}, \mathbf{H}_1^\alpha] \times \nabla(G_2 - G_1) + \\ &+[\mathbf{n}, \mathbf{H}^S] \times \nabla G_2 + [\mathbf{n}, \mathbf{E}_1^\alpha](\sigma_2^* G_2 - \sigma_1^* G_1) + [\mathbf{n}, \mathbf{E}^S]\sigma_2^* G\Big\}\, dS + \\ &+ \int_L \{[\mathbf{n}, \mathbf{H}_1^\alpha] \times \nabla G_1 + \sigma_1^*[\mathbf{n}, \mathbf{E}_1^\alpha]G_1\}\, dL.\end{aligned} \tag{2}$$

$$G_{1,2} = -\frac{e^{ik^*|\mathbf{r}-\mathbf{r}'|}}{4\pi|\mathbf{r} - \mathbf{r}'|} \tag{3}$$

Relations (1) and (2) are the equations of the TIP for a monochromatic field (relative to the boundary S). The material properties of the anamalous region are assumed to be parameters; i.e. the solution of the TIP holds for various values $\sigma_2, \varepsilon_2, \mu_2$. The result is an equivalent family of bodies that generate the same electrical or magnetic field.

If modified Stratton-Chu integrals are used for a monochromatic field, it is possible to derive simpler equations not containing the normal components of $\mathbf{E}$ and $\mathbf{H}$ under the integral:

$$\begin{aligned}\mathbf{E}_1^\alpha(\mathbf{r}') = \nabla' \times \nabla' \times \int_S &\left\{[\mathbf{n}, \mathbf{H}_1^\alpha]\left(\frac{G_2}{\sigma_2^*} - \frac{G_1}{\sigma_1^*}\right) + \frac{G_2}{\sigma_1^*}[\mathbf{n}, \mathbf{H}^S]\right\} ds + \\ +\nabla' \times \int_S &\left\{[\mathbf{n}, \mathbf{E}_1^\alpha](G_2 - G_1) + G_2[\mathbf{n}, \mathbf{E}^S]\right\} ds,\end{aligned} \tag{4}$$

$$\begin{aligned}\mathbf{H}_1^\alpha(\mathbf{r}') = \tfrac{1}{i\omega}\nabla' \times \nabla' \times \int_S &\left\{[\mathbf{n}, \mathbf{E}_1^\alpha]\left(\frac{G_2}{\mu_2} - \frac{G_1}{\mu_1}\right) + \frac{G_2}{\mu_2}[\mathbf{n}, \mathbf{E}^S]\right\} ds + \\ +\nabla' \times \int_S &\left\{[\mathbf{n}, \mathbf{H}_1^\alpha](G_2 - G_1) + G_2[\mathbf{n}, \mathbf{H}^S]\right\} ds.\end{aligned} \tag{5}$$

Equations (1), (2), (4), (5) are all TIP equations (relative to the boundary S) for a monochromatic field. In their numerical solution, it is possible to use the algorithm formulated in Martyshko (1986a).

The case $\sigma_2 = \infty$ (T is a perfect conductor) is of special interest. Then, at the boundary, we have

$$(\mathbf{H}_1, \mathbf{n}) = 0, \ [\mathbf{E}_1, \mathbf{n}] = 0 \tag{6}$$

There are TIP functional equations for determining the surface of a perfect conductor (Martyshko, 1986a) and can be a good starting approximation in solving Eqs (1), (2), (4), (5). In Zhdanov (1988), the theory of Stratton-Chu integrals for a monochromatic field is generalized to inhomogeneous media. Specifically, a magnetically homogeneous medium with a piecewise-continuous distribution of conductivity $\sigma^*(\mathbf{r})$ is considered, and the region T is bounded by a smooth surface S. An expression was derived for the EM field on the basis of its values at the surface using Green's electromagnetic tensors:

$$\mathbf{E}(\mathbf{r}') = \int_S \{i\omega\mu\hat{\mathbf{G}}^e(\mathbf{r}'|\mathbf{r})[\mathbf{n}, \mathbf{H}] + \tilde{\Delta} \times \hat{\mathbf{G}}^e(\mathbf{r}'|\mathbf{r})[\mathbf{n}, \mathbf{E}]\}ds, \tag{7}$$

$$\mathbf{H}(\mathbf{r}') = \int_S \{i\omega\mu\hat{\mathbf{G}}^m(\mathbf{r}'|\mathbf{r})[\mathbf{n}, \mathbf{H}] + \tilde{\Delta} \times \hat{\mathbf{G}}^m(\mathbf{r}'|\mathbf{r})[\mathbf{n}, \mathbf{E}]\}ds, \tag{8}$$

where $\hat{\mathbf{G}}^e, \hat{\mathbf{G}}^m$ are tensor functions of electrical and magnetic types, and $\mathbf{r}' \in T$. If $\mathbf{r}' \in CT$, the integrals in Eqs. (7) and (8) are equal to zero.

In an arbitrary medium, determination of $\hat{\mathbf{G}}^e$ and $\hat{\mathbf{G}}^m$ is difficult. However, for some models, such as a layered medium, this problem can be solved, and accordingly it is possible to write the TIP equation.

Also it is possible to derive a TIP equations for a quasi-stationary field. In this case equation have the form

$$\mathbf{E}_1^a(\mathbf{r}',t') = \int_{-\infty}^{t'}\int_S \Big\{[\mathbf{n},\mathbf{E}_1^a]\times\nabla(G_2^d - G_1^d) + [\mathbf{n},\mathbf{H}_1^a]\left(\mu_2\frac{\partial G_2^d}{\partial t} - \mu_1\frac{\partial G_1}{\partial t}\right) + \\ +[\mathbf{n},\mathbf{E}^S]\times\nabla G_2^d + \mu_2[\mathbf{n},\mathbf{H}^S]\frac{\partial G_2^d}{\partial t} + \\ +(\mathbf{n},\mathbf{E}_1^a)\left[\nabla\left(\frac{\sigma_1}{\sigma_2}G_2^d - G_1^d\right)\right] + \frac{\nabla G_2}{\sigma_2}[\eta + \sigma_1(\mathbf{n},\mathbf{E}^S)]\Big\}\,ds\,dt, \tag{9}$$

$$\mathbf{H}_1^a(\mathbf{r}',t') = \int_{-\infty}^{t'}\int_S \Big\{(\mathbf{n},\mathbf{H}_1^a)\left[\nabla\left(\frac{\mu_1}{\mu_2}G_2^d - G_1^d\right)\right] + \frac{\mu_1}{\mu_2}(\mathbf{H}^S,\mathbf{n})\nabla G_2^d + \\ +[\mathbf{n},\mathbf{H}_1^a]\times\nabla(G_2^d - G_1^d) + [\mathbf{n},\mathbf{E}_1^a](\sigma_2^* G_2^d - \sigma_1^* G_1^d) + \\ +[\mathbf{n},\mathbf{H}^S]\times\nabla G_2^d + [\mathbf{n},\mathbf{E}^S]\sigma_2 G^d\Big\}\,ds\,dt,\ \mathbf{r}' \in CT. \tag{10}$$

3 Numerical examples

As a result of inverse problem solving we obtain the bodies stellate relative to some point with different values of conductivity (permability) wich generated the same (electrical or magnetic) field. We have obtained some numerical examples ($\mu_1 = \mu_2 = \mu_0$, $\mu_0 = 4\pi\cdot 10^{-7}$ H/m.). Figure 1, 2 shows the cross sections cut off by the coordinate planes on TIP solutions of Helmholts equation for function $\mathbf{E}_1^a = (Q_{1x}\frac{e^{ik_2 r_1}}{r_1} + Q_{2x}\frac{e^{ik_2 r_2}}{r_2} + Q_{3x}\frac{e^{ik_2 r_3}}{r_3},\ Q_{1y}\frac{e^{ik_2 r_1}}{r_1} + Q_{2y}\frac{e^{ik_2 r_2}}{r_2} + Q_{3y}\frac{e^{ik_2 r_3}}{r_3},\ Q_{1z}\frac{e^{ik_2 r_1}}{r_1} + Q_{2z}\frac{e^{ik_2 r_2}}{r_2} + Q_{3z}\frac{e^{ik_2 r_3}}{r_3})$, $\mathbf{E}^S(P) = (Q_{4x}\frac{e^{ik_1 r_4}}{r_4},\ Q_{4y}\frac{e^{ik_1 r_4}}{r_4},\ Q_{4z}\frac{e^{ik_1 r_4}}{r_4})$, $P_1, P_2, P_3 \in T^+$, $P_4, P \in T^-$, $r_i = |PP_i|$, $i = \overline{1,4}$, $P_4(0,0,8)$, $Q_{1x} = Q_{1y} = Q_{1z} = 1$, $Q_{2x} = Q_{2y} = Q_{2z} = 2$, $Q_{3x} = Q_{3y} = Q_{3z} = -3$, $Q_{4x} = Q_{4y} = Q_{4z} = -9$.

1. Figure 1 shows numerical results solving TIP for case of halfspace for $\sigma_1/\sigma_2 = 1/10$, $\sigma_1/\sigma_2 = 1/100$, $\omega = 4\pi\cdot 10000$.

2. Figure 2 shows numerical results solving TIP for $\omega = 4\pi\cdot 10000$, $\omega = 4\pi\cdot 1000$, $\omega = 4\pi\cdot 100$, $\sigma_1/\sigma_2 = 1/5$.

4 References

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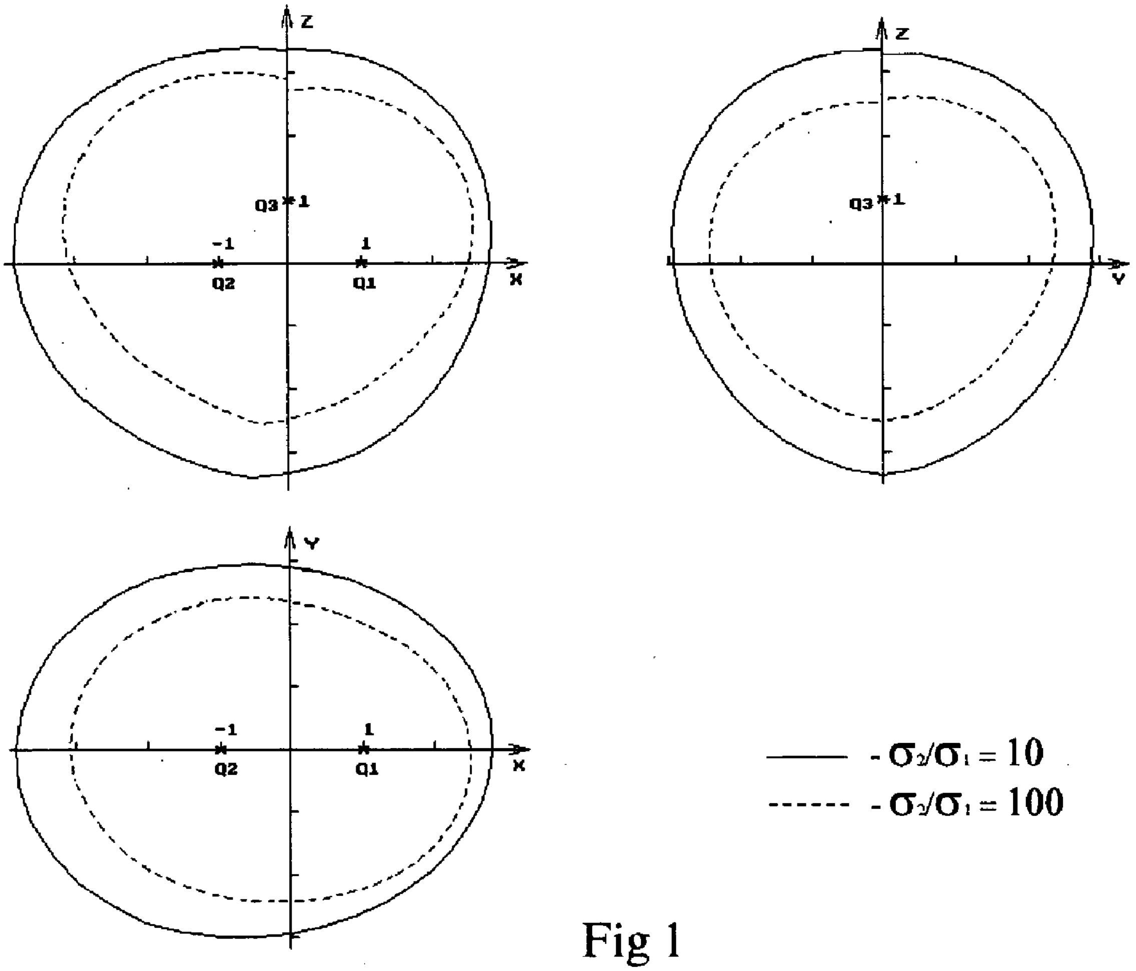
Z
X
Y
Q3
Q2
Q1
-1
1
- $\sigma_2/\sigma_1 = 10$
- $\sigma_2/\sigma_1 = 100$

Fig 1

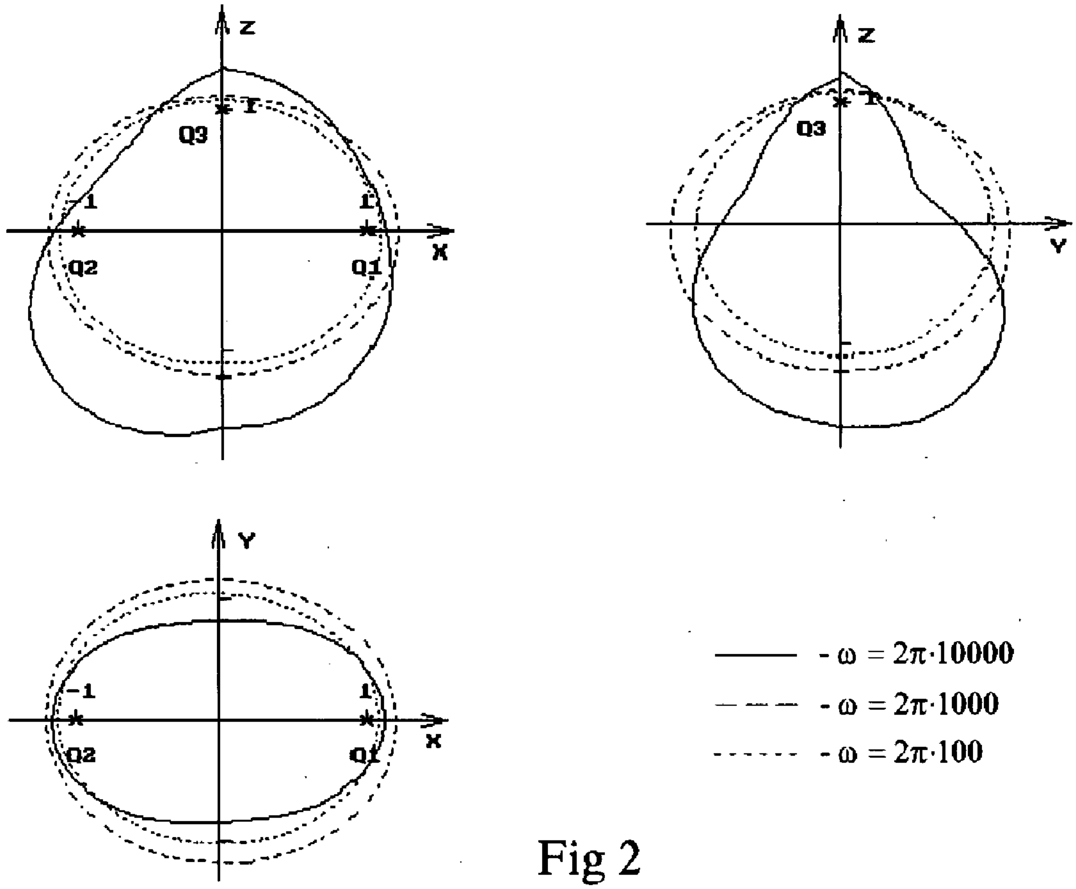
Z
X
Y
Q3
Q2
Q1
-1
1
- $\omega = 2\pi\cdot10000$
- $\omega = 2\pi\cdot1000$
- $\omega = 2\pi\cdot100$

Fig 2